

171 COPY

2

AD-A234 758

FTD-ID(RS)T-1276-90

## FOREIGN TECHNOLOGY DIVISION

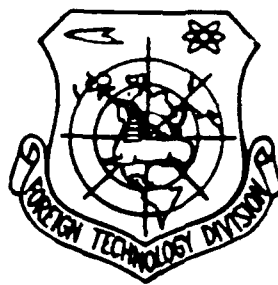


APPLICATION OF TRANSFORMATIONAL IDEAS TO  
AUTOMATIC FLIGHT CONTROL DESIGN

by

Li Wenhua, Liang Feng

171 COPY  
171 COPY  
171 COPY



Approved for public release;  
Distribution unlimited.



91 4 10 028

# HUMAN TRANSLATION

FTD-ID(RS)T-1276-90 21 March 1991

MICROFICHE NR: FTD-91-C-000245

APPLICATION OF TRANSFORMATIONAL IDEAS TO  
AUTOMATIC FLIGHT CONTROL DESIGN

By: Li Wenhua, Liang Feng

English pages: 13

Source: Acta Aeronautica Et Astronautica Sinica,  
Vol. 11, Nr. 4, 1990, pp. 165-170

Country of origin: China

Translated by: SCITRAN

F33657-84-D-0165

Requester: FTD/TTTAV/Paul Freisthler

Approved for public release; Distribution unlimited.

✓

By	
Dissem	
Avail	
Dist	
A-1	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION

PREPARED BY

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WPAFB OHIO

#### GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

**TITLE:** APPLICATION OF TRANSFORMATIONAL IDEAS TO  
AUTOMATIC FLIGHT CONTROL DESIGN

**AUTHOR:** Li Wenhua and Liang Feng

**SUMMARY** To opt to use geometrical methods to design actual nonlinear systems, one must, first of all, resolve the problems of the complexity of transformations, the great amount of calculations, and other similar problems. Because of this, simplifying the design process is extremely important. In this article, based on transformational ideas, we have obtained two methods and used them in the design of the U.S. F-8 Crusader fighter plane's vertical control system. The simulation results clearly demonstrate that the new control laws are obviously superior to the original nonlinear optimization control laws. Moreover, it is possible to guarantee that aircraft can make high angle attack flights.

**Key Words** transformation, nonlinear system, feedback design

## I. INTRODUCTION

Due to the excellence of linear system design tools, at the present time, relevant nonlinear systems design theory depends for help in every case on the design methods of linear systems. In the theory of differential geometry, transformation methods are based on this idea. The key to this is nothing other than--in looking for a type of transform--taking the original system transform to be a linear system or quasilinear system in a different space. After doing that, it is possible, in the new space, to opt for the use of linear system design tools to carry out the design. Because almost all transformations can be recognized as differential homeomorphisms, and differential homeomorphism guarantees that the structural characteristics of systems before and after transformations is invariable, in this way, going again through reverse transformations, one obtains the original system control laws which can cause the original system pole points not to follow the operating points in their changes. The difficulty with this method is in getting the transformational relationships. Actual or practical transformation forms general have a base transformation and a feedback transformation. Transformation methods can, thus, be of various types

and kinds.

There are those systems whose nonlinear natures are due to an inappropriate selection of base. Naturally, it is possible to opt for the use of overall transformations, taking them and changing them into standard forms of integral patterns that can be controlled. Hunt and others in Reference (1), on the basis of solutions to linear partial differential equations, gave out sufficient conditions for overall transformations as well as general methods of constructing them. Polynumeric actual or real systems are only capable of partial transformation into linear systems. Su R., in Reference (2), proved or confirmed the sufficient conditions for the existence of partial differential homeomorphisms. As far as the size of the partial domain for this origin point is concerned, there still is no conclusion. Furthermore, the general run of systems are only capable of transforming into linear systems within a tangent space. In this regard, there is Baumann along with others who brought up extended linearization (3) and Reboulet and others who brought up

B166

pseudolinearization(4). The two methods are basically in agreement with each other. Even if one gets the closed ring characteristic values set up when the closed ring operations are at different equilibrium points to be invariable, because that is so, one avoids point to point linearization.

From Reference (5), it is possible to see that general forms of high order system transformation relationships are always extremely difficult to obtain. There are only block triangular form systems that are an exception. Because of this, designers, when designing automatic aircraft control systems, always take flight equations and do simplification processing in order to obtain block triangular forms. However, this necessarily carries with it a certain error. What follows deals with investigations into the F-8 fighter plane's vertical control system and how transformations were selected for use to arrive at control rules.

## II. THE SET UP OF VERTICAL MOVEMENT EQUATIONS FOR AIRCRAFT

Modern generations of high performance aircraft normally fly within large envelopes or ranges, and nonlinear factors severely impact the dynamic responses of systems. A common example is high angle of attack flight in fighter planes. Their lift coefficients are certainly not capable of using linear angle of attack functions for their precise expression. At high angular velocities of roll and turn nonlinear inertial combinations lead to loss of stability. In order to improve dynamic response, it is necessary, on the basis of nonlinear models, to carry forward the design work.

In Reference 6, one takes lift coefficients and uses third degree angle of attack functions to simulate actual lift curves. Cosine functions take the initial two terms of a Taylor expansion, at the same time taking into consideration all the many nonlinear factors, under conditions in which aircraft have no acceleration, the Mach number is 0.85, the altitude of flight is 9000m, and the condition is level flight. One goes through a trimming down process and obtains equations for the vertical movement of the F-8 fighter plane (let the angle of attack  $\alpha = x_1$ , let the pitch angle  $\theta = x_2$ , let the pitch velocity  $\dot{\theta} = \dot{x}_3$ , and, let the rudder deviation angle be  $\delta_e$ ).

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -0.877x_1 + 0.47x_1^3 + 3.846x_1^5 - 0.019x_1^7 - x_1^9 x_3 - 0.088x_1x_3 + x_3 \\ x_3 \\ -4.208x_1 - 0.47x_1^3 - 3.564x_1^5 - 0.396x_3 \end{pmatrix} + \begin{pmatrix} -0.215 \\ 0 \\ -20.967 \end{pmatrix} \delta_e \quad (1)$$

In Reference (6), in consideration of the linear optimization control law ( $\delta_e = \mu_0$ ):

$$\mu_0 = -0.053x_1 + 0.5x_2 + 0.521x_3$$

When the initial perturbation value for the angle of attack  $\alpha_0 > 0.43 \text{ rad} (24.5^\circ)$  the system loses stability. Because of this, one finds the third order form for nonlinear optimization control

$$\mu = \mu_0 + 0.04x_1^2 - 0.048x_1x_2 + 0.374x_1^2 - 0.312x_1^2x_2 \quad (2)$$

(2) This equation comes from solving Hamilton-Jacobi equations. The equation in question can only guarantee  $\dot{V}$  ( $V$  is the Lyapunov function) in its negative specification. It does not guarantee the positive specification of  $V$ . Because this is true, when the initial perturbation value  $\alpha_0 > 0.48 \text{ rad} (27.5^\circ)$ , (1) is still unstable. Below, we opt for the use of a new method of transformation to solve for even more satisfactory results.

### III. TWO TYPES OF TRANSFORMATION METHODS

#### 1. Method 1

Consider the system

$$\dot{x}(t) = f(x(t)) + g(x(t)) \cdot u(t) \quad (3)$$

In this, analyzing  $f, g \in R^n$  in the domain or region adjacent to the origin,  $f(0)=0$ . It can be seen that  $f$  and  $g$  make the vector field  $e^\infty$  including the origin open up on  $R^n$ . Our objective is to search for the linear independent transformation  $T(x)$ :

$R^n \rightarrow R^n$  that will take system (3) and change it into the type of integral form of quasicontrollable standard form.

B167

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \vdots \\ \dot{T}_n \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{n+1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} u \quad (4)$$

In this,  $T_{n+1}$  should be the arbitrary function  $T_1, \dots, T_n$ . It is easy to see that.

$$\dot{T} = \frac{\partial T}{\partial x} \cdot \dot{x} = \frac{\partial T}{\partial x} f + \frac{\partial T}{\partial x} g \cdot u$$

Taking the equation above and comparing it with (4), it is possible to get

$$T_{i+1} = \frac{\partial T_i}{\partial x} f = \langle dT_i, f \rangle \quad i=1, 2, \dots, n-1$$

$$\frac{\partial T_i}{\partial x} g = \langle dT_i, g \rangle = 0 \quad i=1, 2, \dots, n-1$$

Therefore, one has

$$\begin{aligned} \langle dT_1, g \rangle &= \langle d \langle dT_1, f \rangle, g \rangle \\ &= -\frac{\partial T_1}{\partial x} (\text{ad}^1 f, g) = 0 \\ &\vdots \\ \langle dT_{n-1}, g \rangle &= (-1)^{n-2} \frac{\partial T_1}{\partial x} (\text{ad}^{n-2} f, g) = 0 \\ \langle dT_n, g \rangle &= (-1)^{n-1} \frac{\partial T_1}{\partial x} (\text{ad}^{n-1} f, g) = 1 \end{aligned} \quad (5)$$

In this  $(\text{ad}^i f, g) = \langle \langle \text{ad}^i f, g \rangle, g \rangle$

Let the controllable array  $\mathcal{A} = [g, (\text{ad}^1 f, g), \dots, (\text{ad}^{n-1} f, g)]$ .

Then, equation (5) can be generalized to be

$$\frac{\partial T_1}{\partial x} \mathcal{A} = [0, 0, \dots, 0, 1]$$

If  $\mathcal{A}$  is nonsingular; select  $\mathcal{A}^{-1}$ 's final line. Assume it is  $q(x)$  ( $q(x)$  is a  $1 \times n$  dimensional vector quantity). Then, it is possible to obtain



$$\partial T_1 / \partial x = q(x) \quad (6)$$

If  $q(x)$  is a gradient field, then (6) is a soluble equation. If this is not the case, then, normally, it is possible to place a function  $\sigma(x)$ , causing  $\tilde{q}(x) = \sigma(x) \cdot q(x)$  to be a gradient field. Then, from  $\partial T_1 / \partial x = \tilde{q}(x)$  it is possible to solve for  $T_1(x)$ . At this time, system (3) transforms to become like the form below

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \vdots \\ \dot{T}_n \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{n+1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ (-1)^{n-1} \sigma(x) \end{pmatrix} u$$

Taking this method and using it on system (1), it is first necessary to solve the controllable array  $\mathcal{A}$

$$\mathcal{A} = [g, (ad^1 f, g), (ad^2 f, g)]$$

In this

$$\begin{aligned} (ad^1 f, g) &= -\partial f / \partial x \cdot g \\ (ad^2 f, g) &= -\frac{\partial(\partial f / \partial x \cdot g)}{\partial x} f + \left( \frac{\partial f}{\partial x} \right)^2 g \end{aligned}$$

After that, it is still necessary to solve for  $\delta^{-1}$ . Also, solving equation (6), the amount of calculations can be imagined. In a practical study of system(1): the influence of  $\delta$  on the system is only  $-0.019x_2^2$  of the  $\alpha$  signal. Due to the fact that  $\delta$  is a slowly transforming modality, its connection with and influence on the quickly changing modalities of  $\alpha$  and  $\beta$  are very small. Because this is the case, we would be well served if we first carried out designing of the interior ring composed of the quickly changing modalities

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -0.877x_1 + 0.47x_1^2 + 3.846x_1^3 - x_1^2x_2 - 0.088x_1x_3 + x_3 \\ -4.208x_1 - 0.396x_2 - 0.47x_1^2 - 3.564x_1^3 \end{bmatrix} \\ &+ \begin{bmatrix} -0.215 \\ -20.967 \end{bmatrix} \delta, \\ &= \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} -0.215 \\ -20.967 \end{bmatrix} \delta, \end{aligned} \quad (7)$$

System (7) 's Jacob array or matrix is

$$\begin{aligned} F = \frac{\partial f}{\partial x} &= \begin{bmatrix} -0.877 + 0.94x_1 + 11.538x_1^2 - 2x_1x_2 - 0.088x_2 & +0.088x_1 - x_1^2 \\ -4.208 - 0.94x_1 - 10.7x_1^2 & -0.396 \end{bmatrix} \\ \text{Let } F &= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \\ (ad^1 f, g) &= -\frac{\partial f}{\partial x} g = \begin{bmatrix} a_1(x) \\ a_2(x) \end{bmatrix} \end{aligned}$$

In this

$$a_1(x) = 0.215F_{11} + 20.967F_{12}, \quad a_2(x) = 0.215F_{21} + 20.967F_{22}$$

Solving  $\delta^{-1}$ 's final line, one obtains

$$q(x) = \frac{1}{20.967a_1(x) - 0.215a_2(x)} [20.967 - 0.215]$$

Taking  $c(x) = 20.967a_1(x) - 0.215a_2(x)$

it is possible to obtain  $T_1 = 20.967x_1 - 0.215x_2$

Because

$$T_1 = (dT_1, f) = \frac{\partial T_1}{\partial x} f = 20.967f_1 - 0.215f_2$$

$$\dot{T}_1 = (20.967F_{11} - 0.215F_{21})\dot{x}_1 + (20.967F_{12} - 0.215F_{22})\dot{x}_2$$

From  $\dot{T}_2$ , it is possible to obtain

$$T_2 = (20.967F_{11} - 0.215F_{21})f_1 + (20.967F_{12} - 0.215F_{22})f_2$$

Since this is true, system (7)'s status feedback rule is

$$u = (-b_1T_1 - b_2T_2 - T_3)/(-\sigma(x))$$

$$= (b_1T_1 + b_2T_2 + T_3)/\sigma(x) \quad (3)$$

In this,  $b_1$  and  $b_2$  are closed ring type multiterm equation characteristic coefficients, that is, when  $e_1$  and  $e_2$  are closed ring pole points, one has  $(s-e_1)(s-e_2) = s^2 + b_1s + b_2$ .

## 2. Method 2

Consider the second order affine nonlinear system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} + \begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix} u \quad (9)$$

It is always capable of transforming into a block triangular one.

Because, in the most general situation, when  $g_1(x) \cdot g_2(x) \neq 0$ , equation (9) can be written to be

$$\begin{bmatrix} g_1 \dot{x}_1 - g_1 \dot{x}_1 \\ g_1 \dot{x}_1 \end{bmatrix} = \begin{bmatrix} f_1 g_1 - f_1 g_1 \\ f_1 g_1 \end{bmatrix} + \begin{bmatrix} 0 \\ g_1 g_1 \end{bmatrix} u \quad (10) \quad B169$$

Let  $\dot{\tilde{x}}_1 = g_2 \dot{x}_1 - g_1 \dot{x}_2$ . Then, equation (10) can be written to become a block triangle form. As far as its overall or global transformation is concerned, it is possible to let

$$\dot{\tilde{T}}_1 = g_2 \dot{x}_1 - g_1 \dot{x}_2$$

Then,

$$T_1 = \int_0^{x_1} g_2 dx_1 - \int_0^{x_1} g_1 dx_1$$

Because

$$\dot{T}_2 = \dot{\tilde{T}}_1 = f_1 g_2 - f_2 g_1$$

therefore

$$\dot{\tilde{T}}_2 = p(x) \cdot \dot{x}_1 + q(x) \cdot \dot{x}_2$$

In this

$$p(x) = \frac{\partial f_1}{\partial x_1} g_2 + f_1 \frac{\partial g_2}{\partial x_1} - \frac{\partial f_2}{\partial x_1} g_1 - f_2 \frac{\partial g_1}{\partial x_1}$$

$$q(x) = \frac{\partial f_1}{\partial x_2} g_1 + f_1 \frac{\partial g_2}{\partial x_2} - \frac{\partial f_2}{\partial x_2} g_1 - f_2 \frac{\partial g_1}{\partial x_2}$$

If one writes it to become a controllable standard form of expression, then one has

$$\begin{aligned} T_3(x) &= p(x) \cdot f_1 + q(x) \cdot f_2 \\ \sigma(x) &= -(p(x) g_1 + q(x) \cdot g_2) \end{aligned}$$

In the same way, it is possible to obtain the status feedback rule or law

$$u = (-b_1 T_1 - b_2 T_2 - T_3) / (-\sigma(x)) \quad (11)$$

If one takes system (7) and substitutes into the various equations discussed above, it is possible to see that equation (11) and equation (8) are of the same type.

#### IV. VERTICAL FEEDBACK CONTROL LAWS AND SIMULATION RESULTS

As far as system (1) is concerned, due to the fact that, after the placement of the pole points of small sealed rings, there is no relationship with the operating points, as a result, it is necessary to stabilize the integral signal  $\delta$  of  $\vartheta$ . It is only necessary, after the control law of small sealed rings to attach a linear feedback  $K_{(\text{unclear})}$   $\vartheta$ . As a result of this, one obtains the vertical control law

$$\delta_s = (b_1 T_1 + b_2 T_2 + T_3) / \sigma(x) + K_s \vartheta \quad (12)$$

In this  $T_1$ ,  $T_2$ ,  $T_3$ , and  $\sigma(x)$  have definitions which can be seen in the sections above.

If one takes the interior ring pole points and positions them at  $(-2,0)$  and  $(-4,0)$ , and, again, on the basis of the quality of actual and simulated responses, selects  $K_{(\text{unclear})} = 0.1$ , and, below, if one takes control law (2) and control law (12) and makes use of them to make a complete comparison of the responses after the original system, the results of simulations are as shown below:

1. When the original perturbation  $\alpha_0 = 0.25 \text{ rad} (14.3^\circ)$ , the responses of the two angles of attack  $\alpha$  are closely in line with each other, however, due to the fact that the initial rudder offset angles were not the same, control law (12) was slightly better (see Fig.1).

2. When  $\alpha_0 = 0.47\text{rad}(27^\circ)$ , the response of control law (12) is clearly superior to control law (2) (see Fig.2).

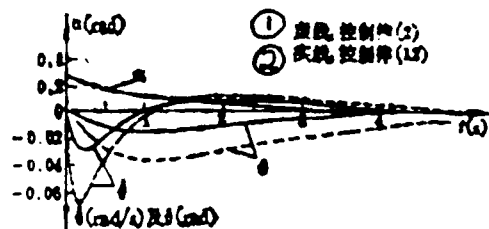


Fig.1

A Comparison of Responses When  $\alpha_0 = 0.25\text{rad}$  (1) Dotted line control Law (2) (2) Solid line control law(12)

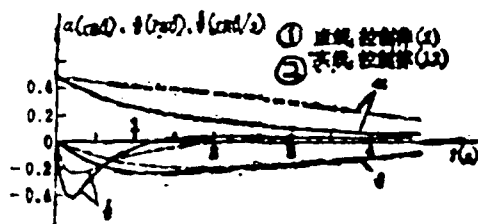


Fig.2

A Comparison of Responses When  $\alpha_0 = 0.47\text{rad}$  (1) Dotted line Control Law (2) (2) Solid line Control Law (12)

3. After  $\alpha_0 > 0.47\text{rad}$ , control law (2) begins to cause systems to diverge. However, straight up to the time when  $\alpha_0 = 0.6\text{rad}(34.4^\circ)$  the response of control law (12) is still good (see Fig.3).

B170

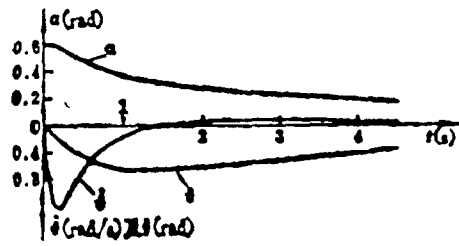


Fig.3 Responses of Control Law (12) When  $\alpha_0 = 0.6\text{rad}$

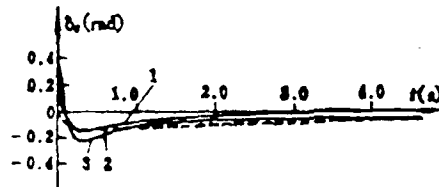


Fig.4 A Comparison of Rudder Offset Angle Response Curves (Curve 1 represents the rudder offset for control law (12) when  $\alpha_0 = 0.47\text{rad}$ . Curve 2 represents the rudder offset for control law (2) when  $\alpha_0 = 0.47\text{rad}$ . Curve 3 represents the rudder offset for control law (12) when  $\alpha_0 = 0.6\text{rad}$ )

## V. SEVERAL POINTS IN CONCLUSION

This article, based on the transformation thinking in differential geometry, designed for the vertical control systems of the F-8 fighter plane nonlinear computer control laws or rules. The

results of simulations clearly show that these control laws are capable of guaranteeing that the aircraft can fly at high angles of attack. The dynamic qualities are much better than those of the original nonlinear optimization controls. A practical analysis of the rudder offset responses in Fig.4 is capable of showing one that, when angles of attack are relatively large, due to  $\partial C_L / \partial \alpha < 0$  ( $C_L$  is the lift coefficient), there are problems with the aircraft fighting the controls. However, control law (12) still actualizes feedback to defeat this before it happens, and, as a result, it avoids this fighting of the controls.

Speaking in terms of aircraft in general, the vertical direction is due to the separate characteristics of  $\theta$  and  $\alpha$ , as well as  $\dot{\theta}$ . Because this is true, the two methods that have been discussed above both are feasible. As far as high order nonlinear systems are concerned, if it is possible to set up laws to take them and transform them into block triangular forms, then, both of the two methods are made simpler and easier.

## REFERENCES

- 1 Hunt L, Su R, Meyer G, Global Transformation of Nonlinear Systems, IEEE Trans. AC, 1983; 28, (1)24—31
- 2 Su R, On the Linear Equivalent of Nonlinear Systems, Syst. Contr, 1982; 2, 48—52
- 3 Baumann W, Rugh W, Feedback Control Systems by Extended Linearization, IEEE. Trans. AC, 1986; 31, (1)40—45.
- 4 Reboulet C, Champetier C, A New Method for Linearization Nonlinear Systems; the Pseudolinearization, Int. J. Control, 1984; 40, (4)633—638
- 5 Stanilow H, Carl A, State-feedback Control of Nonlinear Systems, Int. J. Control, 1986; 43, (6) 1497—1514.
- 6 Garrad W, Jordan J, Design of Nonlinear Automatic Flight Control Systems, Automatica, 1977; 13, 497—505



DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>
CS09 BALLISTIC RES LAB	1
CS10 R&T LABS/AVEADCOM	1
CS13 ARRADCOM	1
CS35 AVRADCOM/TSARCOM	1
CS39 TRASANA	1
Q591 FSTC	4
Q619 MSIC REDSTONE	1
Q008 NTIC	1
E053 HQ USAF/INET	1
E404 AEDC/DOF	1
E408 AFWL	1
E410 AD/IND	1
F429 SD/IND	1
P005 DOE/ISA/DDI	1
P050 CIA/OCR/ADD/SD	2
AFTT/LDE	1
NOTC/OIC-9	1
CCV	1
MIA/PIS	1
LLYL/CODE L-309	1
[REDACTED]	1
NSA/T513/TDL	2
ASD/FTD/TTIA	1
FSL	1